

Lepton Number Violation in Supersymmetric Grand Unified Theories

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Abstract

We argue that the nature of the global conservation laws in Supersymmetric Grand Unified Theories is determined by the basic vacuum configuration in the model rather than its Lagrangian. It is shown that the suppression of baryon number violation in a general (R -parity violating) superpotential can naturally appear in some extended $SU(N)$ SUSY GUTs which, among other degenerate symmetry-breaking vacua, have a missing VEV vacuum configuration giving a solution to the doublet-triplet splitting problem. We construct $SU(7)$ and $SU(8)$ GUTs where the effective lepton number violating couplings immediately evolve, while the baryon number non-conserving ones are safely projected out as the GUT symmetry breaks down to that of the MSSM. However at the next stage, when SUSY breaks, the radiative corrections shift the missing VEV components to some nonzero values of order M_{SUSY} , thereby inducing the ordinary Higgs doublet mass, on the one hand, and tiny baryon number violation, on the other. So, a missing VEV solution to the gauge hierarchy problem leads at the same time to a similar hierarchy of baryon vs lepton number violation.

1 Introduction

The Standard Model (SM), while being extremely successful in describing interactions of quarks and leptons at low energies, still has many unanswered questions. Among these one problem is predominant: the unification of all elementary forces within the framework of a simple gauge theory and the ensuing hierarchy of mass scales in particle physics at large and small distances. Possible solutions to this problem are commonly related to supersymmetry (SUSY) [1] and Grand Unified Theories (GUT) [2]. At present they receive some indirect (largely qualitative) experimental support from the apparent lightness of the Higgs boson, the values of gauge couplings given by precision measurements and the heavy top quark mass. At low energies the SUSY GUT turns into the Minimal Supersymmetric Standard Model (MSSM). Of course the MSSM can certainly be considered on its own as a simple SUSY extension of the Standard Model, leaving aside for the moment the question of unification.

However, no matter which level of theory is considered, there is one point that crucially distinguishes SUSY from non-SUSY models. This is that they do not contain the automatic accidental symmetries, corresponding to baryon (B) and lepton (L) number conservation, which are present in the ordinary Standard Model. Does this mean that B and L number can be violated in the SUSY context or must some special protecting symmetry be postulated in the MSSM? Usually, the requirement of B and L number conservation in the MSSM is indeed satisfied by postulating the existence of some multiplicative discrete symmetry called R-parity [1]. An exact R-parity (*RP*) implies that SUSY particles should be produced in pairs and that the lightest SUSY particle (LSP) is stable.

On the other hand, there is no fundamental reason to prefer models with exact *RP* over those with broken *RP* in the framework of the supersymmetric SM, where not only fermions but also their scalar superpartners automatically become the carriers of lepton and baryon numbers. Thereby, among the basic renormalisable couplings in the low-energy MSSM superpotential, one would generally expect to find the lepton and baryon number violating ones

$$\Delta W = \mu_i L_i H_u + \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k. \quad (1)$$

Here, i, j, k are generation indices and a summation is implied (colour and weak isospin indices are suppressed); $L_i(Q_j)$ denote the lepton (quark) $SU(2)$ -doublet superfields and $\overline{E}_i(\overline{U}_i, \overline{D}_i)$ are $SU(2)$ -singlet lepton (*up*-quark, *down*-quark) superfields; μ_i are mass parameters which mix lepton superfields with the *up*-type Higgs superfield H_u , while λ_{ijk} ($\lambda_{ijk} = -\lambda_{jik}$), λ'_{ijk} and λ''_{ijk} ($\lambda''_{ijk} = -\lambda''_{ikj}$) are dimensionless couplings. The first three terms in (1) violate lepton number, while the last violates baryon number.

While SUSY-inspired B number violation (BNV) leads in general to unacceptably fast proton decay and must be highly suppressed, SUSY-inspired L number violation (LNV) could readily occur at a level consistent with present experimental constraints, but large enough for the observation of some of its spectacular manifestations [3] at present or future colliders. Remarkably enough, instead of *RP*, another (gauge) discrete symmetry could appear in the MSSM: superstring-inherited Z_3 baryon parity [4], which strongly protects B number and allows for L number violation only. Thus, as long as it is not in conflict with any phenomenology, SUSY-inspired lepton number violation merits further detailed investigation, both theoretically and experimentally.

From the theoretical point of view, the principal question concerns the search for a Grand Unified framework within which, while treating quarks and leptons equally, L-violation should

be allowed at the same time as B-conservation. Unfortunately, the discrete symmetries acceptably protecting B-conservation while allowing L-violation in the MSSM, such as the above mentioned Z_3 baryon parity, transform quarks and leptons differently and, thereby, are incompatible with the known GUTs. Nevertheless several extended GUT models have been constructed [5], where the coexistence of lepton number violation and baryon number conservation can in principle be arranged. This is typically achieved by introducing at the Planck scale high-dimensional operators, involving Higgs and matter multiplets in some exotic representations of the underlying GUT symmetry, and then imposing additional custodial symmetries to ensure that only the required set of LNV high-order operators is allowed. These operators become the renormalisable LNV couplings (1) at lower energies after the GUT symmetry breaks at the GUT scale M_{GUT} .

Despite some progress, one has the uneasy feeling that such a solution to this problem looks rather artificial, as it is generically correlated neither with the nature of the GUT nor with its breaking pattern. Instead, we suggest that it is just the breaking pattern of the underlying GUT symmetry which could give a fundamental reason for the difference in treatment of the baryon and lepton numbers of the matter particles involved in GUTs. We show that a suppression of baryon number violating interactions in the superpotential (1) naturally occurs in some $SU(N)$ SUSY GUTs where a missing VEV vacuum configuration develops, which also gives a solution to the doublet-triplet splitting problem [6]. We construct explicit examples of RP -violating $SU(7)$ and $SU(8)$ GUTs where the effective LNV couplings immediately evolve from the GUT scale, while the baryon number non-conserving ones are safely projected out by the missing VEV vacuum configuration breaking the GUT symmetry down to that of the MSSM. However, at the next stage when SUSY breaks, radiative corrections shift the missing VEV to some nonzero value of order M_{SUSY} and induce BNV violating couplings with hierarchically small coupling constants $\lambda''_{ijk} = O(M_{SUSY}/M_{GUT})$, which appear to be of phenomenological interest [3].

2 Missing VEV solutions in $SU(N)$ GUTs

The most elegant solution to the gauge hierarchy problem in supersymmetric $SU(N)$ GUTs could well be related to the existence of a missing VEV vacuum configuration [7], according to which the basic adjoint scalar Σ_j^i ($i, j = 1, \dots, N$) does not develop a VEV in some of the directions in $SU(N)$ space. Through its coupling with a pair of Higgs fields H and \bar{H} , their masses are split in a hierarchical way so as to have light weak doublets breaking electroweak symmetry and giving masses to up and down quarks, on the one hand, and superheavy colour triplets mediating proton decay, on the other. However, it is well known [7] that a missing VEV solution can not appear in $SU(N)$ GUTs in the ordinary one-adjoint scalar case. This is due to the presence of a cubic term Σ^3 in the general Higgs superpotential W leading to the unrealistic trace condition $Tr\Sigma^2 = 0$ for the missing VEV vacuum configuration, unless there is a special fine-tuned cancellation between $Tr\Sigma^2$ and driving terms stemming from other parts of the superpotential W .

2.1 The two-adjoint alternative

So, it seems the only way to obtain a natural missing VEV solution in $SU(N)$ theories is to exclude the cubic term Σ^3 from the superpotential, by imposing some extra reflection symmetry

on the adjoint supermultiplet Σ

$$\Sigma \rightarrow -\Sigma \quad (2)$$

On its own the elimination of the Σ^3 term leads to the trivial unbroken symmetry case. However the inclusion of higher even-order Σ terms (supposedly inherited from superstrings or induced by gravitational corrections) in the effective superpotential leads to an all-order missing VEV solution, as was shown in recent papers [6]. Alternatively one can introduce another adjoint scalar Ω with only renormalisable couplings appearing in W .

Let us consider briefly the high-order term case first. The $SU(N)$ invariant superpotential for an adjoint scalar field Σ conditioned also by the gauge Z_2 reflection symmetry (2)

$$W_A = \frac{1}{2}m\Sigma^2 + \frac{\lambda_1}{4M_P}\Sigma^4 + \frac{\lambda_2}{4M_P}\Sigma^2\Sigma^2 + \dots \quad (3)$$

contains, in general, all possible even-order Σ terms scaled by inverse powers of the (conventionally reduced) Planck mass $M_P = (8\pi G_N)^{-1/2} \simeq 2.4 \cdot 10^{18}$ GeV. It is readily shown that the necessary condition for any missing VEV solution to appear in the $SU(N) \otimes Z_2$ invariant superpotential W_A is the tracelessness of all the odd-order Σ terms

$$Tr\Sigma^{2s+1} = 0, \quad s = 0, 1, 2, \dots \quad (4)$$

This condition uniquely leads to a missing VEV pattern of the type

$$<\Sigma> = \sigma Diag(\overbrace{0 \dots 0}^{N-k}, \overbrace{1 \dots 1}^{k/2}, \overbrace{-1 \dots -1}^{k/2}), \quad (5)$$

where the VEV value σ is calculated using the Σ polynomial taken in W_A (3). The vacuum configuration (5) gives rise to a particular breaking channel of the $SU(N)$ GUT symmetry

$$SU(N) \rightarrow SU(N-k) \otimes SU(k/2) \otimes SU(k/2) \otimes U(I)_1 \otimes U(I)_2, \quad (6)$$

which we will discuss in some detail a little later. So we conclude from Eqs. (5, 6) that a missing VEV solution could actually exist, with the ordinary MSSM gauge symmetry $SU(3)_C \otimes SU(2)_W \otimes U(1)_Y$ surviving at low energies, provided that $N \geq 7$.

The superpotential (3) can be viewed as an effective one, following from an ordinary renormalisable two-adjoint superpotential with the second heavy adjoint scalar integrated out. Hereafter, although both approaches are closely related, we deal for simplicity with the two-adjoint case. So let us consider a general $SU(N)$ invariant renormalisable superpotential for two adjoint scalars Σ and Ω , also satisfying the gauge-type Z_2 reflection symmetry ($\Sigma \rightarrow -\Sigma, \Omega \rightarrow \Omega$) inherited from superstrings:

$$W_A = \frac{1}{2}m\Sigma^2 + \frac{1}{2}M_P\Omega^2 + \frac{1}{2}h\Sigma^2\Omega + \frac{1}{3}\lambda\Omega^3. \quad (7)$$

Here the second adjoint Ω can be considered as a state originating from a massive string mode with the Planck mass M_P . The basic adjoint Σ may be taken at another well motivated scale $m \sim M_P^{2/3}M_{SUSY}^{1/3} \sim O(10^{13})$ GeV [8] where, according to many string models, the adjoint moduli states $(1_c, 1_w), (1_c, 3_w)$ and $(8_c, 1_w)$ (in a self-evident $SU(3)_C \otimes SU(2)_W$ notation) appear. In the present context these states can be identified as just the non-Goldstone remnants Σ_0, Σ_3 and Σ_8 of the relatively light adjoint Σ which breaks $SU(N)$ in some way. However, all

our conclusions remain valid for any reasonable value of m , which is the only mass parameter (apart from M_P) in the model considered.

As a general analysis of the superpotential W_A (7) shows [6], there are just four possible VEV patterns for the adjoint scalars Σ and Ω : (i) the trivial unbroken symmetry case, $\Sigma = \Omega = 0$; (ii) the single-adjoint condensation, $\Sigma = 0$, $\Omega \neq 0$; (iii) the "parallel" vacuum configurations, $\Sigma \propto \Omega$ and (iv) the "orthogonal" vacuum configurations, $Tr(\Sigma\Omega) = 0$. The Planck-mass mode Ω , having a cubic term in W_A , in all non-trivial cases develops a standard "single-breaking" VEV pattern

$$\langle \Omega \rangle = \omega Diag(\overbrace{1 \dots 1}^{N-k}, \underbrace{-\frac{N-k}{k} \dots -\frac{N-k}{k}}_k), \quad (8)$$

which breaks the $SU(N)$ GUT symmetry to

$$SU(N) \rightarrow SU(N-k) \otimes SU(k) \otimes U(I). \quad (9)$$

However, in case (iv), the basic adjoint Σ develops the radically new missing VEV vacuum configuration (5), thus giving a "double breaking" of $SU(N)$ to (6). Using the approximation $\frac{h}{\lambda} \gg \frac{m}{M_P}$, which is satisfied for any reasonable values of the couplings h and λ in the generic superpotential W_A (7), the VEV values are given by

$$\omega = \frac{k}{N-k} \frac{m}{h}, \quad \sigma = \left(\frac{2N}{N-k} \right)^{1/2} \sqrt{m M_P / h} \quad (10)$$

respectively. Surprisingly, just the light adjoint Σ develops the largest VEV in the model which, for a properly chosen adjoint mass m and coupling constant h , can easily come up to the string scale M_{str} (see [9]).

Furthermore, as concluded above, one must consider $SU(N)$ GUTs with $N \geq 7$, in order to have the standard gauge symmetry $SU(3)_C \otimes SU(2)_W \otimes U(1)_Y$ remaining after the breaking (6). As is easily seen from Eqs. (5, 6), there are two principal possibilities: the weak-component and colour-component missing VEV solutions respectively. If it is granted that the "missing VEV subgroup" $SU(N-k)$ in (6) is just the weak symmetry group $SU(2)_W$, as is traditionally argued [7], one is led to $SU(8)$ as the minimal GUT symmetry ($N-k = 2, k/2 = 3$) [6]. Another, and in fact the minimal, possibility is to identify $SU(N-k)$ with the colour symmetry group $SU(3)_C$ in the framework of an $SU(7)$ GUT symmetry ($N-k = 3, k/2 = 2$) [9]. The higher $SU(N)$ GUT solutions, if considered, are also based on just those two principal possibilities: the weak-component or colour-component missing VEV vacuum configurations respectively.

Let us see now how this missing VEV mechanism works to solve the doublet-triplet splitting problem in $SU(8)$ or $SU(7)$ GUT with the superpotential W_A (7). It is supposed that there is a reflection-invariant coupling of the ordinary MSSM Higgs-boson containing supermultiplets H and \bar{H} with the basic adjoint Σ , but not with Ω , in the superpotential W_H

$$W_H = f_0 \bar{H} \Sigma H + W'_H \quad (\Sigma \rightarrow -\Sigma, \bar{H} H \rightarrow -\bar{H} H) \quad (11)$$

The second part W'_H contains possible mixings with other scalar fields, which are inessential for the moment. The superfields H and \bar{H} do not develop VEVs during the first stage of the symmetry breaking (6). Thereupon the first term in W_H turns into a mass term for H and

\bar{H} determined by the missing VEV pattern (5). This vacuum, while giving generally heavy masses (of the order of M_{GUT}) to H and \bar{H} , leaves their weak components strictly massless. To be certain of this, we must specify the multiplet structure of H and \bar{H} for both the weak-component and the colour-component missing VEV vacuum configurations, that is in $SU(8)$ and $SU(7)$ GUTs respectively. In the $SU(8)$ case H and \bar{H} are fundamental octets whose weak components (ordinary Higgs doublets) do not get masses from the basic coupling (11). In the $SU(7)$ case H and \bar{H} are 2-index antisymmetric 21-plets in which, after projecting out the extra heavy states (see Section 3.1), there is left just one pair of massless Higgs doublets as a consequence of the coupling (11). Thus, there is a natural doublet-triplet splitting in both cases and we also have a vanishing μ term at this stage. However, radiative corrections generate a μ term of the right order of magnitude at the next stage when SUSY breaks [6].

2.2 Projection to low energies

Missing VEV vacua, which ensure the survival of the MSSM at low energies, only appear in $SU(N)$ GUTs with a higher symmetry group than the standard $SU(5)$ model. In order not to spoil gauge coupling unification, the extra gauge symmetry should also be broken, $SU(N) \rightarrow SU(5)$, at the GUT scale. Then the following question arises: how can the missing VEV survive this extra symmetry breaking with at most a shift of order the electroweak scale? This requires, in general, that the superpotential (7) be strictly protected from any large influence from the $N - 5$ scalars φ^k ($k = 1, \dots, N - 5$) providing the extra symmetry breaking (or from uncontrollable gravitational corrections). Technically, such a custodial symmetry may be a superstring-inherited anomalous $U(1)_A$ [10], which can naturally keep two sectors of the total superpotential separate and then induce a high-scale extra symmetry breaking through the Fayet-Iliopoulos (FI) D -term [11]:

$$D_A = \xi + \sum Q_A^k |\langle \varphi^k \rangle|^2, \quad \xi = \frac{\text{Tr}Q_A}{192\pi^2} g_{\text{str}}^2 M_P^2. \quad (12)$$

Here the sum runs over all "charged" scalar fields in the theory, including those which do not develop VEVs and which contribute to $\text{Tr}Q_A$ only. For realistic or semi-realistic models, $\text{Tr}Q_A$ has turned out to be quite large, $\text{Tr}Q_A = O(100)$ (see [12] for a recent discussion). Therefore, the spontaneous breaking scale of the $U(1)_A$ symmetry and of the related extra gauge symmetry is naturally located at the string scale. The protecting anomalous $U(1)_A$ symmetry is needed to keep the scalars $\varphi^{(k)}$ and $\bar{\varphi}^{(k)}$ essentially decoupled from the basic adjoint superpotential W_A (7), so as not to strongly influence its missing VEV vacuum configuration (5). Otherwise potentially dangerous couplings could appear of the type $\bar{\varphi}^{(k)} \Sigma \varphi^{(k)}$, where the $\varphi^{(k)}$ and $\bar{\varphi}^{(k)}$ scalar superfields are taken in pairs of conjugate fundamental representations (N and \bar{N}) of $SU(N)$. If these couplings actually appeared, they would give rise to shifts in the missing VEV components of the adjoint scalar Σ_B^A of the order $\Sigma_B^A \sim \frac{\delta_B^A}{m} \bar{\varphi}^{(k)} \varphi^{(k)} \sim O(M_{GUT})$, as directly follows from the minimisation condition for the scalar potential. So the presence of a protecting symmetry is essential for the missing VEV mechanism to function properly.

We will now enlarge on this key point in order to gain a better understanding of the missing VEV approach. The symmetry protected separation of the adjoint scalar and the φ^k scalar sectors in the total superpotential implies the appearance of an accidental global symmetry $SU(N)_{\Sigma-\Omega} \otimes U(N)\varphi$ in the $SU(N) \otimes U(1)_A$ gauge theory considered. This global symmetry is in turn radiatively broken, resulting in a set of pseudo-Goldstone (PG) states of the type

$$5 + \bar{5} + SU(5) - \text{singlets} \quad (13)$$

which gain a mass at the TeV scale where SUSY softly breaks [6]. There can be a maximum of $N - 5$ families of PG states of the type (13), corresponding to the case where the scalars $\varphi^{(k)}$ and $\bar{\varphi}^{(k)}$ are only allowed to appear in the Higgs potential through the basic $SU(N)$ and $U(1)_A$ D -terms. In this case the $U(N)\varphi$ global symmetry is increased to $U(N)_{\varphi^{(1)}} \otimes \dots \otimes U(N)_{\varphi^{(N-5)}}$. This case would occur if the $U(1)_A$ charges of the bilinears $\bar{\varphi}^k \varphi^{k'}$ were all positive (or negative), so that they could not appear in the $SU(N) \otimes U(1)_A$ invariant superpotential in any order.

However, in a properly extended model it is possible for the adjoint and fundamental scalar sectors in the superpotential to overlap without disturbing the adjoint missing VEV configuration. This naturally occurs when the scalars $\varphi^{(k)}$ are conditioned by the $U(1)_A$ symmetry to develop orthogonal VEVs along the "extra" directions

$$\varphi_A^{(k)} = \delta_{A,5+k} V_k, \quad k = 1, \dots, N - 5 \quad (14)$$

As a result, some safe non-diagonal couplings $\bar{\varphi}^{(m)} \Sigma \varphi^{(n)}$ are generated between the two sectors, giving contributions to the pseudo-Goldstone masses which leave only one light PG family (12).

Let us consider this possibility in some detail. The least restrictive choice of such safe mixing terms for the general $SU(N)$ case is achieved by introducing two sets of new singlet scalar superfields fields, S_{mn} and T_{mn} , with non-diagonal couplings of the type

$$W_{mix} = \sum_{m < n}^{N-5} \bar{\varphi}^{(m)} [a_{mn} S_{mn} + b_{mn} T_{mn} \Sigma] \varphi^{(n)} \quad (15)$$

which are also invariant under the reflection symmetry $\Sigma \rightarrow -\Sigma$, $T_{mn} \rightarrow -T_{mn}$. The coupling constants a_{mn} and b_{mn} are all of order $O(1)$ and $O(1/M_P)$ respectively, and the $(N-5)(N-6)/2$ singlet scalars S_{mn} and T_{mn} ($m < n$) get their VEVs through the FI D -term (12), as do all the φ and $\bar{\varphi}$ scalars. One can consider the fields S_{mn} as the basic carriers of the $U(1)_A$ charges Q_{mn} which are all taken positive in the model (the fields T_{mn} carry the same charges Q_{mn}). The $U(1)_A$ charges of the $\bar{\varphi}^{(m)} \varphi^{(n)}$ bilinears ($m < n$) appearing in W_{mix} are then determined to be $-Q_{mn}$, while the charges of all the other bilinears, diagonal $\bar{\varphi}^{(m)} \varphi^{(m)}$ and non-diagonal $\bar{\varphi}^{(n)} \varphi^{(m)}$, can always be chosen positive. This implies that any terms containing φ and $\bar{\varphi}$ scalars can only appear in the superpotential if they also include the bilinears $\bar{\varphi}^{(m)} \varphi^{(n)}$ so as to properly compensate the $U(1)_A$ charges. However, for a vacuum configuration where the orthogonality conditions $\bar{\varphi}^{(m)} \varphi^{(n)} = 0$ naturally arise, such terms do not lead (in any order) to the dangerous $\bar{\varphi}^{(k)} \Sigma \varphi^{(k)}$ couplings, although they can contribute to the pseudo-Goldstone masses. In fact these orthogonality conditions are satisfied at the SUSY invariant global minimum of the Higgs potential, as follows from the vanishing F -terms of the superfields S_{mn} (T_{mn}), $\bar{\varphi}^{(m)}$ and $\varphi^{(n)}$ involved in (15):

$$\bar{\varphi}^{(m)} \varphi^{(n)} = 0, \quad a_{mn} S_{mn} = -b_{mn} T_{mn} \Sigma_{5+n}^{5+n}, \quad m < n \quad (16)$$

(no summation is implied). Here the orthogonal VEV values of the scalars $\varphi^{(n)}$ (14) have been used. One can now readily see that non-diagonal mass terms appear for the PG states related to the multiplets $\bar{\varphi}^{(m)}$ and $\varphi^{(n)}$

$$M_{mn} \equiv [W''_{mix}]_{\bar{\varphi}^{(m)} \varphi^{(n)}} = b_{mn} T_{mn} (\Sigma - \Sigma_{5+n}^{5+n} \cdot I), \quad m < n \quad (17)$$

where I is the $N \times N$ unit matrix. Diagonalisation of the mass matrix (17) explicitly shows that one PG superposition $5 + \bar{5}$ (13) is left massless, while the others become heavy¹. This is in fact

¹This mass matrix is in fact a "triangular" $(N - 5) \times (N - 5)$ matrix with zeros on the main diagonal, $M_{mn} = 0$ for $m \geq n$. Such a matrix has in general only one zero eigenvalue.

a general consequence of the symmetry breaking pattern involved. The point is that neither of the other mass-terms M_{mm} and M_{nm} can be allowed by $U(1)_A$ symmetry for any generalisation of the superpotential W_{mix} (15). Otherwise the dangerous $\overline{\varphi}^{(k)} \Sigma \varphi^{(k)}$ couplings inevitably appear as well. So, one can conclude that even in the general case one PG family of the type (13) always exists. Together with the ordinary quarks and leptons and their superpartners these PG states, both bosons and fermions, determine the particle spectrum at low energies. In most of what follows the existence of just one family of PG states at the sub-TeV scale will be assumed.

We consider below both of the minimal possible GUTs, $SU(7)$ and $SU(8)$, with the missing VEV solution naturally allowing the survival of the MSSM down to low energies. Whereas the $SU(7)$ model is taken as an ordinary one-family unifying GUT [9], the $SU(8)$ model can include unification of the quark-lepton families as well [13].

3 One-family unifying GUT: $SU(7)$

By analogy with the standard $SU(5)$ model, we take the simplest anomaly-free set of matter fields, consisting of the combination of the fundamental and 2-index antisymmetrical representations of the $SU(7)$ gauge group

$$\left[2\overline{\Upsilon}^A + \overline{\Psi}^A + \Psi_{[AB]} \right]_i \quad (18)$$

($A, B = 1, \dots, 7$ are the $SU(7)$ indices) for each of the three quark-lepton families or generations ($i = 1, 2, 3$). The quarks and leptons belong to the multiplets $\overline{\Psi}^A(\overline{7}) + \Psi_{[AB]}(21)$, while the extra multiplets $\overline{\Upsilon}^A$ are specially introduced in (18) for anomaly cancellation.

There is also a set of Higgs superfields among which are the two already mentioned adjoint Higgs multiplets Σ_B^A and Ω_B^A , responsible for the breaking (6, 9) of $SU(7)$, and a conjugate pair of multiplets $H_{[AB]}$ and $\bar{H}^{[AB]}$ (being the 21-plets of the $SU(7)$) where the ordinary electroweak doublets H_u and \bar{H}_d reside. Besides, as in the general $SU(N)$ case (see Section 2.2), there should be extra-symmetry breaking scalar superfields $\varphi^{(p)}$ and $\overline{\varphi}^{(p)}$ ($p = 1, 2$) which are taken to be fundamental septets and anti-septets respectively. They are supposed to develop their string-scale order VEVs along the "extra" directions

$$\varphi_A^{(1)} = \delta_{A6} V_1, \quad \varphi_A^{(2)} = \delta_{A7} V_2 \quad (19)$$

only through the (FI) D -term related to the $U(1)_A$ symmetry (12). The protecting anomalous $U(1)_A$ symmetry keeps the φ scalars decoupled from the basic adjoint superpotential W_A (7), so as not to strongly influence the missing VEV solution (5) through dangerous couplings of the type $\overline{\varphi}^{(p)} \Sigma \varphi^{(p)}$.

With the given assignment of matter and Higgs superfields, the particle spectrum at low energies looks as if one had just the standard SUSY $SU(5)$ as a starting GUT symmetry, except that one family of PG states of type (13) appears, when a missing VEV vacuum configuration develops in the $SU(7)$ GUT. With this exception, all the other $SU(7)$ inherited states in matter and Higgs multiplets acquire GUT scale masses due to symmetry breaking, thus completely decoupling from low-energy physics. We demonstrate this for the Higgs sector in the next sub-section.

3.1 Higgs sector

We now show that all the states, except for one pair of weak doublets in the basic Higgs multiplets $H_{[AB]}$ and $\overline{H}^{[AB]}$, become superheavy. Firstly one substitutes the colour-component missing VEV solution, obtained from the general case (5) by setting $N = 7$ and $k = 4$, into the superpotential (11). Superheavy masses are thereby generated for most of the components of the H and \overline{H} multiplets. However, the following states (weak, colour and extra symmetry components are explicitly indicated)

$$H_{w6}, \quad \overline{H}^{w6}, \quad H_{w7}, \quad \overline{H}^{w7}, \quad H_{[cc']}, \quad \overline{H}^{[cc']} \quad (20)$$

still remain massless at this stage of $SU(7)$ symmetry breaking (6). Therefore one of the two pairs of weak doublets in (20), as well as the colour triplets, must further become heavy in order to get the ordinary picture of MSSM at low energies. This happens as a result of mixing H and \overline{H} with the specially introduced heavy scalar supermultiplets $\Phi_{[ABC]}$ and $\overline{\Phi}^{[ABC]}$ (being 35-plets of $SU(7)$) in the basic Higgs superpotential

$$W'_H = f \cdot H \overline{\Phi} \varphi^{(1)} + \overline{f} \cdot \overline{H} \Phi \overline{\varphi}^{(1)} + y \cdot S \overline{\Phi} \Phi, \quad (21)$$

(f , \overline{f} and y are dimensionless coupling constants) when the scalars φ get their VEVs, thus breaking the extra gauge symmetry. The presence of the "conjugated" $\overline{\Phi} - H$ and $\Phi - \overline{H}$ mixings in W'_H could allow the dangerous $\overline{\varphi} \Sigma \varphi$ terms, destroying the missing VEV solution, unless the bilinear term $\overline{\Phi} \Phi$ has nonzero $U(1)_A$ charge. Therefore, this term appears in W'_H together with the singlet scalar superfield S , the basic $U(1)_A$ charge carrier introduced earlier in W_{mix} (15) in a general $SU(N)$ context (for $SU(7)$ there appears only one pair of such singlets, S and T).

It should be clear now that the W'_H couplings (21) will rearrange the mass spectrum of the states (20), so as to leave just one pair of massless weak-doublets as needed for the MSSM. By diagonalising the 2×2 mass matrix for the states $H_{[cc']}$ and $\overline{H}^{[cc']}$ and the double-coloured components $\Phi_{[cc'6]}$ and $\overline{\Phi}^{[cc'6]}$, the mass of the colour triplet components in (20) is found to be of order

$$M_* \sim \frac{f\overline{f}}{y} \frac{<\varphi><\overline{\varphi}>}{S} \sim M_{GUT} \quad (22)$$

where the combination of the primary coupling constants f , \overline{f} and y , can be taken $O(1)$ in general. There is a 3×3 mass matrix for the weak doublet states, corresponding to the mixing of the states H_{w6} , H_{w7} and $\Phi_{[w67]}$ and their "conjugates" \overline{H}^{w6} , \overline{H}^{w7} and $\overline{\Phi}^{[w67]}$ respectively. After diagonalisation this matrix leaves just one pair of weak-doublets H^{w6} and \overline{H}^{w6} strictly massless, while the other pair H_{w7} and \overline{H}^{w7} acquires a mass M_* (22) of order M_{GUT} .

In much the same way all the additional states in the $SU(7)$ matter multiplets (18) become superheavy during the starting GUT symmetry breaking $SU(7) \rightarrow SU(5)$ [9].

3.2 Yukawa couplings

The usual dimension-4 trilinear Yukawa couplings are forbidden by $SU(7)$ gauge invariance. So we suppose that all the generalized Yukawa couplings, the RP -conserving (ordinary up and down fermion Yukawas) as well as the RP -violating ones allowed by the $SU(7) \otimes U(1)_A$

symmetry, are given by a similar set of dimension-5 operators of the form ($i, j, k = 1, 2, 3$ are the generation indices, the $SU(7)$ indices $A, B, C = 1, \dots, 7$ are hereafter omitted):

$$\mathcal{O}_{ij}^{up} = \frac{G_{ij}^u}{M_P} (\Psi_i \Psi_j) (H \varphi^{(2)}) \quad (23)$$

$$\mathcal{O}_{ij}^{down} = \frac{G_{ij}^d}{M_P} (\overline{\Psi}_i \Psi_j) (\overline{H} \varphi^{(1)}) \quad (24)$$

$$\mathcal{O}_{ijk}^{rpv} = \frac{G_{ijk}}{M_P} (\overline{\Psi}_i \Psi_j) (\overline{\Psi}_k \Sigma). \quad (25)$$

Further, substituting the VEVs of the scalars Σ (5) and φ (19) into the basic operators (23–25), one obtains at low energies the effective renormalisable Yukawa and LNV interactions with coupling constants

$$Y_{ij}^u = G_{ij}^u \frac{<\varphi^{(2)}>}{M_P}, \quad Y_{ij}^d = G_{ij}^d \frac{<\varphi^{(1)}>}{M_P}, \quad \Lambda_{ijk} = G_{ijk} \frac{<\Sigma>}{M_P}. \quad (26)$$

At the same time the baryon number non-conserving couplings λ''_{ijk} completely disappear. The crucial point is that the adjoint field Σ develops a VEV configuration with strictly zero colour components (5) in the SUSY limit. When SUSY breaks, radiative corrections will shift the missing VEV components of Σ to nonzero values of order M_{SUSY} , thus inducing the ordinary μ -term of the MSSM, on the one hand, and baryon number violating interactions with hierarchically small coupling constants of the order M_{SUSY}/M_{GUT} , on the other.

The effective dimension-5 interactions (23–25) could be generated by the exchange of some heavy states, such as massive string modes. When generated by the exchange of the same superheavy multiplet (that is a vector-like pair of fundamental septets $7 + \overline{7}$), the resulting operators (24) and (25) have effective coupling constants (26) aligned in flavour space [15]:

$$\Lambda_{ijk} = Y_{ij}^d \cdot \epsilon_k \quad (27)$$

The parameters ϵ_k ($k = 1, 2, 3$) include some known combination of the primary dimensionless coupling constants and a ratio of the VEVs of the scalars Σ and φ . This relation (27) further splits into the ones for charged lepton (cl) and down quark (dq) LNV couplings respectively,

$$\lambda_{ijk} = Y_{ij}^{cl} \cdot \epsilon_k, \quad \lambda'_{ijk} = Y_{ij}^{dq} \cdot \epsilon_k, \quad (28)$$

when evolved from the $SU(7)$ scale down to low energies.

So we see that the possible common origin of all the generalised Yukawa couplings, both RP -conserving and RP -violating, at the GUT scale results in some minimal form of lepton number violation, provided that the appropriate heavy-state mediator exists. As a result, we are driven to a simple picture where the flavour structure, as well as the hierarchies of the trilinear LNV couplings in ΔW (1), are essentially aligned with the down quark and charged lepton mass and mixing hierarchies. At the same time, the effective bilinear LNV terms appear to be generically suppressed by the custodial $U(1)_A$ symmetry (for a detailed exposition see a recent paper [15]).

At low energies, the minimal LNV model presented can be viewed as an alternative to another minimal model based on the MSSM, in which only the bilinear LNV terms $\mu_i L_i H_u$ in ΔW (1) are included [5, 16]. Depending on the $U(1)_A$ charges assigned to the matter and Higgs superfields involved, one can generically obtain at low energies either the bilinear model or the

trilinear one considered here. The bilinear model also leads to LNV-Yukawa coupling alignment, by virtue of which many predictions of both models concerning quark flavour conservation are very similar [15]. However, there are principal differences as well. The point is that the influence of the SUSY soft breaking sector, being predominant for the bilinear model, is quite negligible for the present one. Therefore, the LNV-Yukawa alignment, while appearing in both models, leads in the latter case to distinctive flavour-dependent relations between various LNV processes arising from slepton and squark exchanges (which are basically conditioned by the quark and lepton mass hierarchy) [15]. By contrast, in the bilinear model these processes appear to be essentially determined by W and Z boson exchanges and, as a result, are largely flavour-independent. On the other hand, the bilinear model has a serious problem of extension to the GUT framework. Any such extension leads, together with a lepton mixing with a weak Higgs doublet, to a quark mixing with a colour Higgs triplet, thus inducing baryon number violation as well. The only handle one has to address this problem seems to be the use of electroweak scale masses μ_i in the GUT-symmetry invariant bilinear couplings. Their use would mean that new fine-tuning conditions, besides the ordinary gauge hierarchy one, should be satisfied in a very ad hoc way.

An extended discussion of the properties of the $SU(7)$ GUT, including the solution to the doublet-triplet splitting problem, string scale unification, proton decay, hierarchy of baryon vs lepton number violation and neutrino masses, can be found in our recent paper [9].

4 Three-family unifying GUT: $SU(8)$

It is tempting to treat the extra gauge symmetry in a general $SU(N)$ GUT as a flavour symmetry. If so, according to the particular solution (5) for the weak-component missing VEV configuration, the numbers of fundamental colours and flavours must be equal ($n_C = n_F = k/2$) for any even-order $SU(N)$ group, among which the minimal one is $SU(8)$ ($n_C = n_F = 3$). Thus, in the $SU(8)$ case, the missing VEV configuration requires an additional colour-flavour symmetry: $SU(3)_C \leftrightarrow SU(3)_F$.

Having considered the basic matter superfields (quarks and leptons and their superpartners), the question of whether the above flavour symmetry $SU(3)_F$ is really their family symmetry naturally arises. Needless to say, among many other possibilities, the special assignment treating the families as a fundamental triplet of $SU(3)_F$ is the most attractive. In such a case the anomaly-free set of $SU(8)$ antisymmetric multiplets (in a self-evident notation; $A, B, C = 1, 2, \dots, 8$)

$$6 \cdot \overline{8}^A + \overline{28}^{[AB]} + 2 \cdot 56_{[ABC]} + 70_{[ABCD]} \quad (29)$$

is singled out, if we require that after flavour symmetry breaking only three massless families of ordinary quarks and leptons (and their superpartners) are left as chiral triplets of $SU(3)_F$, stemming from the multiplets

$$\overline{28} = (\bar{5}, \bar{3}) + \dots, \quad 70 = (10, \bar{3}) + \dots \quad (30)$$

The remaining $SU(5) \otimes SU(3)_F$ components, in these as well as in the other multiplets (29), acquire heavy masses of order $M_F \sim M_{GUT}$ ². So, one arrives at the chiral $SU(3)_F$ family

²The special multiplet arrangement (29) was considered before by one of us [13] as a possible basis for the family-unifying $SU(8)$ GUT. Remarkably, the multiplets (29) follow from the unique ("each multiplet - one

symmetry case [18], which leads to a natural conservation of flavour both in the particle and sparticle sectors.

Furthermore, there is a universal see-saw mechanism in the $SU(8)$ model, with heavy intermediate states provided by the multiplets (29), which induces non-trivial fermion mass-matrices with many texture ansätze available. So the observed pattern of quark and lepton masses and mixings can appear once the electroweak $SU(2) \otimes U(1)_Y$ symmetry breaks [13]. At the same time, by analogy with the $SU(7)$ case, see (25), the only RP -violating coupling allowed by $SU(8) \otimes U(1)_A$ symmetry is supposed to be given by the dimension-5 operator

$$\mathcal{O}_{rpv} \propto \frac{1}{M_P} (\bar{\Psi}^{[AB]} \Psi_{[ABCD]} \bar{\Psi}^{[CD']} \Sigma_{D'}^D) \quad (31)$$

Here the matter fields $\bar{\Psi}$ and Ψ belong to the basic multiplets (30). One can see now that the weak-component missing VEV solution for Σ (5), when substituted into the operator \mathcal{O}_{rpv} , leaves only the LNV couplings and projects out the baryon number violating ones. At low energies the surviving effective couplings take the form

$$\lambda \epsilon_{\alpha\beta\gamma} (L^\alpha L^\beta \bar{E}^\gamma + r L^\alpha Q^\beta \bar{D}^\gamma) \quad (32)$$

Here $\alpha, \beta, \gamma = 1, 2, 3$ are the generation indices, belonging to the family $SU(3)_F$ symmetry, and r is a factor giving the relative coupling constant renormalisation after evolution from the $SU(8)$ scale to low energies. So, as in the $SU(7)$ case, one has baryon number conservation at the same time as lepton number violation in the SUSY limit.

Meanwhile, despite their common origin, there is a principal difference between the $SU(7)$ and $SU(8)$ cases. The point is that the basic adjoint Σ moduli mass ratio M_3/M_8 appears, according to the missing VEV vacua (5), to be 2 and 1/2 for $SU(7)$ and $SU(8)$ respectively. As was shown in recent papers [9, 14], this ratio essentially determines the high-energy behavior of the MSSM gauge couplings. In fact it follows that the unification scale in $SU(7)$ is pushed to the string scale [9], while the unification scale in $SU(8)$ ranges, at best, close to the standard unification value [6].

5 Conclusions

The absence of automatic global conservation laws in SUSY theories, in contrast to the Standard Model, is frequently considered as a drawback of supersymmetry. Meanwhile phenomenologically, whereas SUSY-inspired B number non-conservation must be highly suppressed, SUSY-inspired L-number violation could occur at a level large enough for the observation of its many spectacular manifestations [3, 15]. One of these manifestations may be the sizeable atmospheric neutrino oscillations recently reported [19], according to which one of the neutrino species is expected to have a mass at least of order 0.1 eV. That means, in general, the particle content of the MSSM or the minimal $SU(5)$ SUSY GUT should be extended to include new states, that is fundamentally heavy right-handed neutrinos or even light sterile left-handed ones. Neutrino masses per se do not yet give any conclusive evidence in favour of SUSY theories. However sizeable LNV in the charged lepton sector and, of course, in the decays of the lightest supersymmetric particle [15], if actually observed, could qualify as generic SUSY inspired phenomena. In such a situation the following question would arise, which should be addressed within

time") set of $SU(11)$ multiplets [17] after the symmetry breaking $SU(11) \rightarrow SU(8)$ and the exclusion of all the conjugated (under $SU(8)$) multiplets except the self-conjugated one $70_{[ABCD]}$.

the framework of Grand Unification rather than the MSSM: what could stand behind such a tremendous hierarchy of lepton vs baryon number violation?

In this connection we suggested that the nature of the global conservation laws in SUSY theories is determined by the basic vacuum configuration which breaks the underlying GUT symmetry. Following this idea, we have argued that the GUTs with a natural missing VEV solution to the doublet-triplet splitting problem could, simultaneously, provide the reason for treating lepton and baryon number carrying matter fields differently. We have shown that missing VEV vacuum configurations, ensuring the survival of the MSSM gauge symmetry at low energies, only emerge in extended $SU(N)$ GUTs with $N \geq 7$. Further, the one-family unifying $SU(7)$ and the three-family unifying $SU(8)$ GUTs have been constructed. In both cases the effective LNV couplings immediately evolve from the GUT scale, while the baryon number non-conserving ones are safely projected out by the missing VEV vacuum configuration, which breaks the starting GUT symmetry down to that of the MSSM. However, at the next stage when SUSY breaks, radiative corrections shift the missing VEV to some nonzero value of order M_{SUSY} , thus inducing the ordinary μ -term of the MSSM, on the one hand, and BNV couplings with the hierarchically small constants $\lambda''_{ijk} = O(M_{SUSY}/M_{GUT})$, on the other. So, a missing VEV solution to the gauge hierarchy problem leads, in a literal sense, to the same hierarchy of baryon vs lepton number violation.

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